About endurance limit of ductile inhomogeneous materials

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The theory describing the fatigue mechanism in elasto-plastic material containing pores or inclusions has been developed. An attempt at quantitative determination of the effect of endurance limit reduction by analysis of sizes of plastic zones formed near the inclusions, and their cracking has been done. The geometrical configuration, consisting of a round inclusion from which a nucleating crack emerged, was considered, and the stress intensity factor of such configuration was analysed. Based on a threshold value of ΔK below which crack propagation ceases, the critical value of loading stress was determined. Theoretical results were compared with results from experiments, showing quite good agreement.

1. Introduction

Rational design using porous or composite materials requires evaluation of strength of such inhomogenous media in the conditions of complex loading.

The reaction for cyclic load is specially interesting because fatigue usually occurs at lower stress levels than for other forms of destruction, and the majority (about 80%) of practically observed fractures have a fatigue character. The strength criteria of inhomogenous materials essentially are created either from empirical theories based on stress tensor component functions or by building failure process models and analysing them using microfracture apparatus.

One such model is presented in this paper and both the above methods were used first by assuming, that satisfying the plasticity condition in local areas in the vicinity of pores in the presence of cyclic loading will lead to almost instantaneous cracking of these areas, and secondly by analysis of the stability of the structure containing cracks, of the length of the previously existing plastic zones.

Because the fatigue fracture process usually starts from the surface of the element when plastic deformations are facilitated, the problem is restricted to the surface layer and consequently to plane stress conditions. The assumption is also made that inclusions are rare enough so that no interaction between them exists.

2. Model formation

2.1. Plastic regions development

Let us assume that an element of the matrix material plate containing a separated circular pore of diameter D is loaded by stress S acting far from the hole and that the yield stress of the matrix material is Y (see Fig. 1). If $S \times \alpha K > Y$, where αK is the stress concentration coefficient of the inclusion, the plastic zone starts to be formed near the pore. The plastic zone formation in the vicinity of inclusions can be observed on the micro- and macroscales.

At microscale levels, a map of dislocation generations in the matrix material around the cylindrical pores [1] or semicircular dislocation segments in the matrix of the composite close to the interphase boundaries with circular enforcement inclusions [2], can be seen. The quantitative analysis, of the plastic zones ranged around circular holes was performed numerically [3, 4], theoretically [5] and experimentally [6].

The author of this paper also used an approximate solution to analyse a thick ring with a small plastic inclusion. The results are shown in Fig. 1 where the plastic zone range A in relation to the hole diameter D, as a function of applied stress S in relation to yield stress Y of the matrix material, is presented. (The hardening coefficient of the material $H \approx 0$.)

All results indicate a relativity good identity. Unfortunately the theoretical expressions of the solution are complicated and further application is difficult.

To continue analysis, a formula was proposed which approximated very closely the theoretical, numerical and experimental results in the region of interest.

$$\frac{A}{D} = \alpha\left(\frac{S}{Y} - \beta\right)^{\gamma}$$

where α, β, γ are coefficients, with $\alpha = 2.58$; $\beta = 0.333$; and $\gamma = 1.43$. This leads to the relation

$$Y = \frac{S}{\left\{\exp\left[\frac{\ln(A/D) - \ln 2.58}{1.43}\right] + 0.333\right\}}$$

in the region of $0 < A/D < 0.55$ for plastic zone range.

2.2. Fatigue of plastic regions

It is obvious that accumulation of plastic strain energy in a limited volume of material will lead to crack

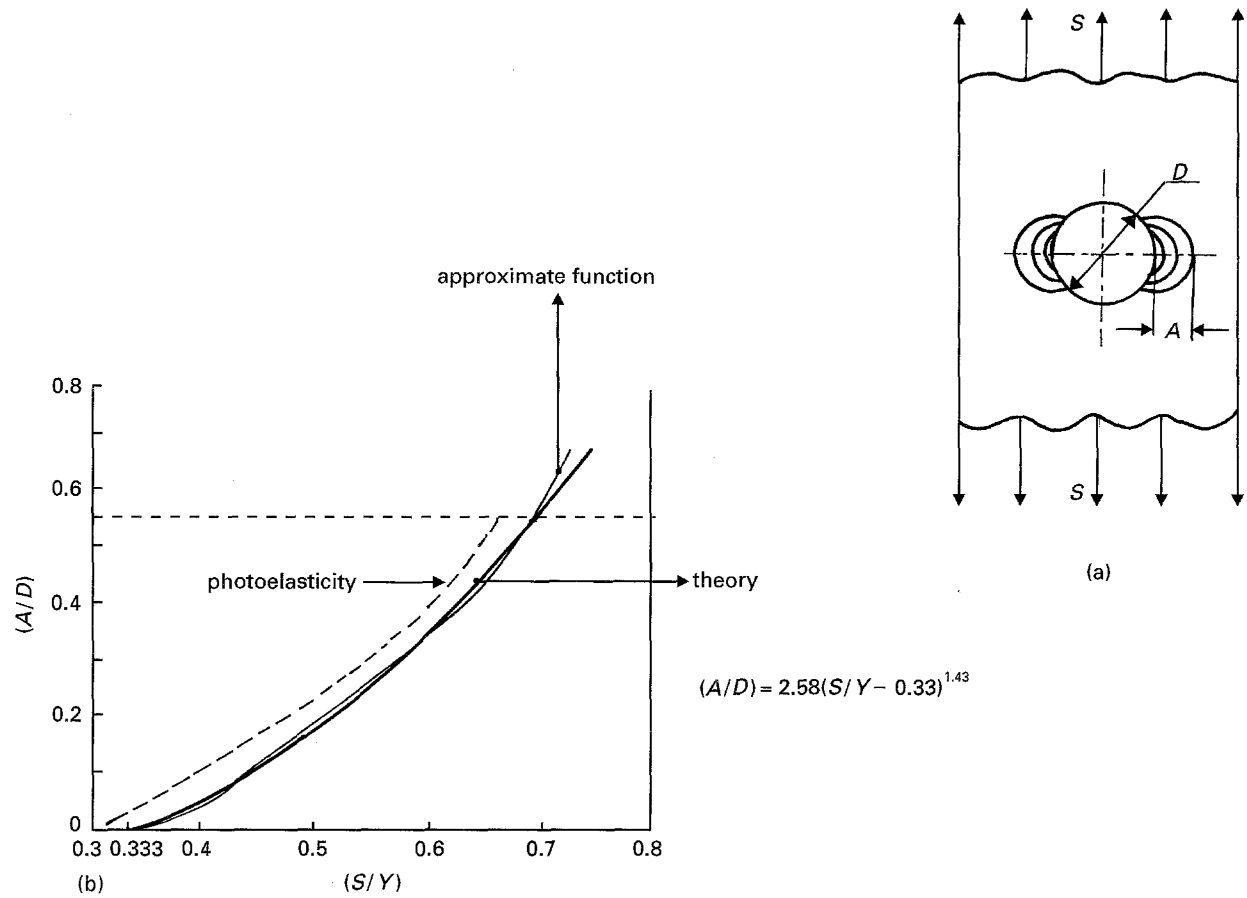

Figure 1 Range of plastic zones (approximate function).

formation. In addition, a plastically deformed region usua lly experiences cyclic strain hardening which facilitates crack propagation. According to Manson's law the fatigue life N_f can be expressed by the plastic strain component $\Delta\varepsilon_p$ as

$$N_f = \left(\frac{\Delta\varepsilon_p}{\varepsilon_f}\right)^{1/\alpha_1}$$

where ε_f and α_1 are coefficients. For example, Manson's point P3 has a coordinate

$$\Delta\varepsilon_p = \frac{1}{4}\left[\ln\left(\frac{1}{1 - RA}\right)\right]$$

where RA is the reduction of sample area. In practice, less than 11% of the plastic deformation corresponds to only 10 cycles. In comparison with numbers of cycles usually considered in fatigue problems the fracture occurs almost instantaneously. Thus by cyclic loading the plastic zones will crack very quickly forming the geometrical configuration presented in Fig. 2a.

2.3. *K* coefficient determination

The stress intensity factor which describes the intensity of all stress components in the crack tip vicinity for the case under consideration was determined by Bowie [7] in his fundamental work. The problem was solved using Mushelisvili's methods and the stress function was assumed to have the form of a polynomial.

The results are presented by Bowie in numerical and graphical form (Fig. 2) where

$$K = S(\pi A)^{0.5} f\left(\frac{A}{D}\right)$$

To enable further analysis, Bowie's results were expressed by the function

$$f\left(\frac{A}{D}\right) = \delta\left(\frac{A}{D}\right)^{\rho} + \theta$$

where δ, ρ, θ are coefficients ($\delta = -2.53$; $\rho = 0.385$; $\theta = 3.36$).

The above formula expressed the original function with acceptable exactness in the region of interest

$$0 < \frac{A}{D} < 0.55$$

Now the stress intensity factor can be expressed as

$$K = S\left[\pi\left(\frac{A}{D}\right)D\right]^{0.5}\left[-2.53\left(\frac{A}{D}\right)^{0.385} + 3.36\right]$$

or

$$K = S\left[\pi D\, 2.58\left(\frac{S}{Y} - 0.333\right)^{1.43}\right]^{0.5}$$

$$\times\left\{-2.53\left[2.58\left(\frac{S}{Y} - 0.333\right)^{1.43}\right]^{0.385} + 0.336\right\}$$

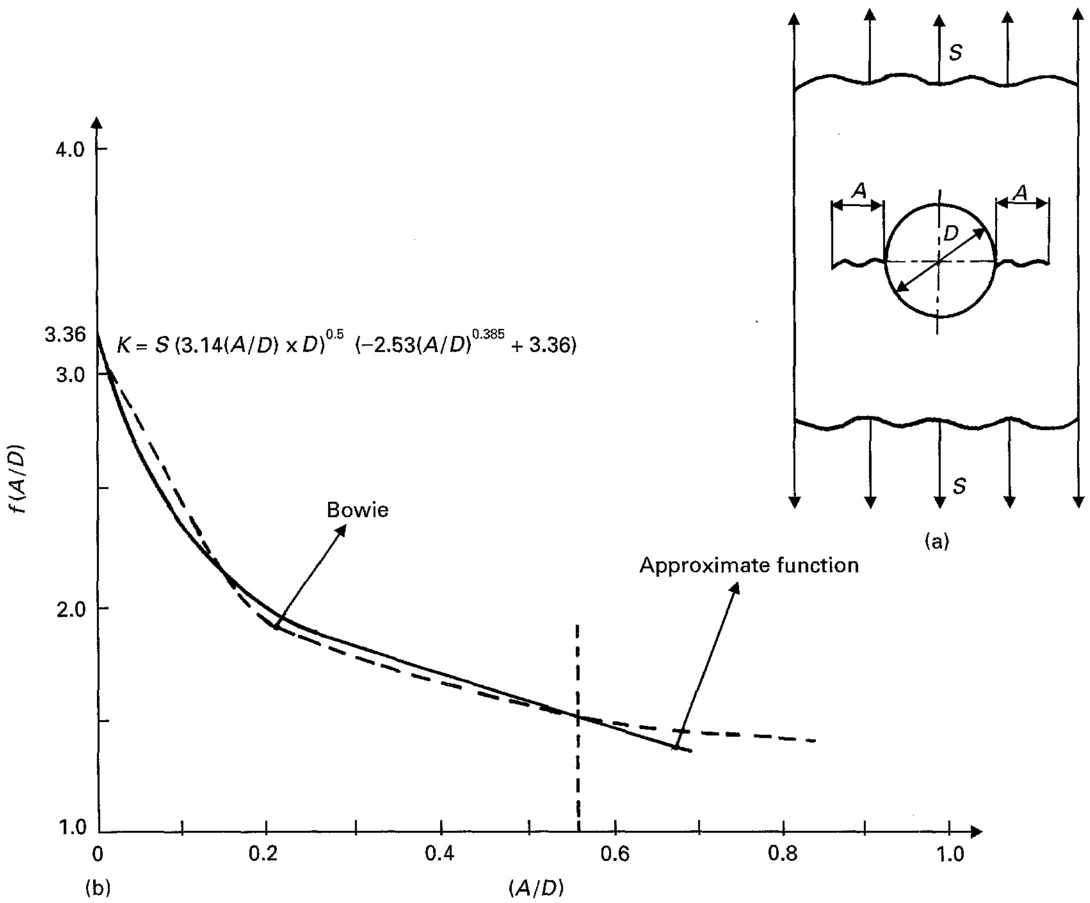

Figure 2 Stress intensity coefficient (approximate function).

2.4. Crack stability condition

The length of cracks in this problem are relatively small. However, according to the Kitagawa–Takahashi concept [8], Fig. 3, they can be treated according to linear elastic fracture mechanics rules. The stress in the considered problem is kept below the limit of 2/3 yield stress of the matrix material and the influence of the plastic zones in the crack tip is small.

It is well known that if oscillation of ΔK of stress intensity factor K does not exceed the threshold value specific for any material marked as KTH the crack propagation will not occur.

Determination of KTH is described by standards and is based on the general principles of fracture mechanics. Thus, the crack stability condition can be formulated as:

$$KTH \geqslant K = S\left[\pi D\alpha\left(\frac{S}{Y}-\beta\right)^{\gamma}\right]^{0.5} \times\left\{S\left[\alpha\left(\frac{S}{Y}-\beta\right)^{\gamma}\right]^{p}+\theta\right\}$$

When this condition is satisfied the crack will not propagate by loading stress equal to S and this level of stress will be noticed as the endurance limit.

The formula gives the possibility to control the influence of:

(a) yield stress of the matrix material,
(b) diameter of the pore, and
(c) threshold value KTH on the endurance limit of the porous material.

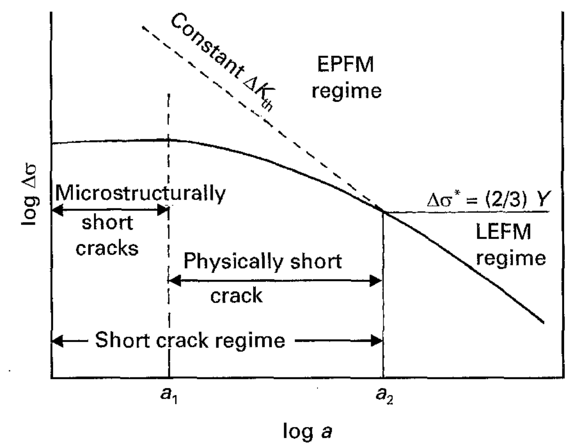

Figure 3 The Kitagawa–Takahashi curve.

3. Intercrystalline cracking, correction for development of fracture surface and correction for deviation of cracking direction

According to our observations fatigue cracking, at least in the starting region, develops in an intercrystalline manner. Grain boundaries are especially weakened regions. Usually impurities are situated here, especially oxides and sulphides and therefore

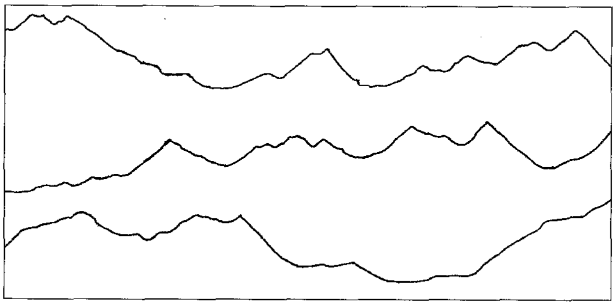

Figure 4 Cracked surface profile.

frequently chains of micropores are formed. The fracture surface corresponds to grain boundaries and the crack propagates in a zigwefdfvzag form in both directions parallel and perpendicular to its front. An example of a cracked surface profile is presented in Fig. 4.

The above developed formula should contain the real value of KTH corresponding with the real zigzag form of the crack in the microscale. Determination of KTH described by standards corddresponds with macroscale conditions and is based on the general principles of fracture mechanics.

The assumption is made that the crack is flat and is always perpendicular to the stress digdg$\beta\beta$rection. So in the macroscale test the "condensated" value of KTH is measured with parallel λ and perpendicular $\mathscr{P}$ coefficients.

The microscale threshold value $KTHM$ is given by

$$KTHM = \frac{KTJ}{\lambda \mathscr{P}}$$

The stress intensity factor obtained from macroscopic examinations on standard specimen is calculated from formula

$$K = \frac{\Delta P}{BW^{1/2}} f\left(\frac{a}{W}\right)$$

where ΔP is the change of loading, W is the height of the specimen, B is the width of the specimen, and a is the crack length.

It is assumed that the width of the cracking front is equal to the width of the specimen B. In fact, the length of the crack front is greater by parallel surface development coefficient λ.

The perpendicular $\mathscr{P}$ factor can be determined on the basis of K coefficient for the zigzag form of the crack.

The surface development coefficient $\psi = 1.45$ indicates that the main inclination of the zigzag development surface is about 45°. The stress intensity for such a zigzag crack was determined by Isida [9] who discovered also that the coefficient in the stress intensity factor formula for K_{I} and K_{II} are almost independent of the zigzag crack length.

Calculation of equivalent KIZ for zigzag gives:

$$KIZ = (K_{\text{I}}^2 + K_{\text{II}}^2)^{0.5} = 0.782S(\pi Na)^{0.5}$$

where Na = real length of the zigzag. $Na \cos 45° = L$ where L = length of the flat crack in the macroscale model. Thus $KIZ = 0.930 = S(\pi L)^{0.5}$.

Comparison with the flat crack used in the macroscale test when $K = 1.12\delta(\pi L)^{0.5}$ gives the perpendicular development coefficient $\mathscr{P} = 0.83$.

Thus $KTHM = KTH\, 0.69 \times 0.83 = 0.572KTH$.

4. Experiments and discussion of the results

The above developed theory was applied to powder metallurgy materials based on iron powder (Höganäs powder NC-100-24).

The maximum pore size which was observed was 0.4 mm in diameter (Fig. 5) so the pores of size 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 (mm) were analysed.

The KTH value determined in the macroscale test was $KTH \approx 200$ (N mm$^{-3/2}$) and this gives $KTHM = 115$ (N mm$^{-3/2}$) for the microscale.

Substituting the above data into the crack stability formulae we can present the family of lines describing

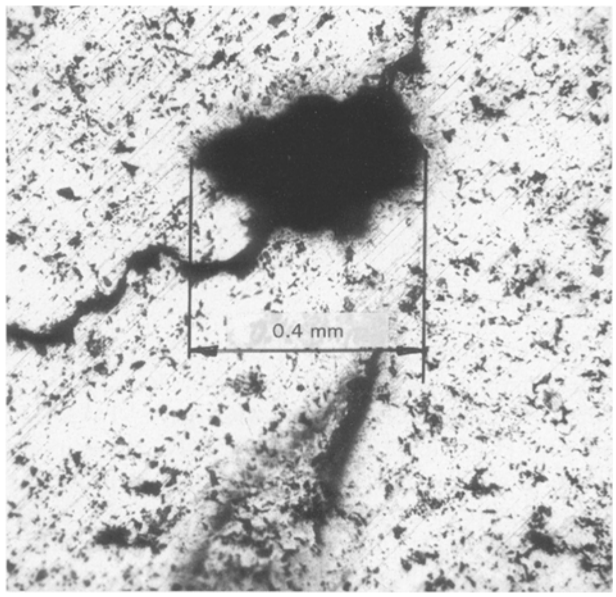

Figure 5 Example of a large pore.

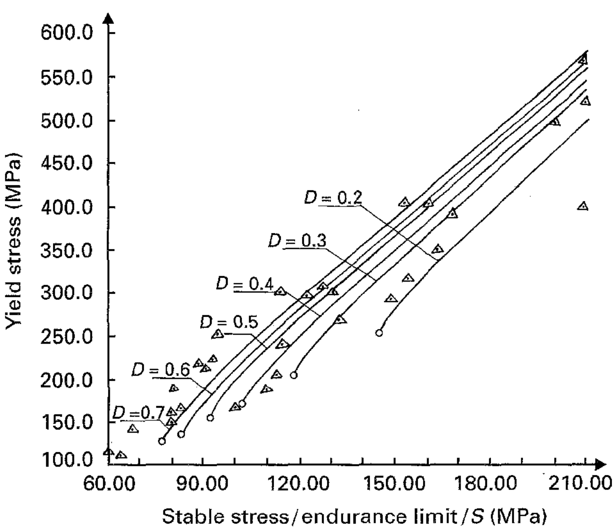

Figure 6 Comparison of theoretical and experimental results. D = hole diameter and ○ refers to end of model application.

the relation between crack stability stress S and yield stress of matrix Y for different inclusion diameters D. It reveals an almost linear relation between S and Y, see Fig. 6.

Such a linear relation is very well known to practicing engineers working with powder metallurgy materials. The diagrqdxczcam presents the experimental results for 26 different sintered materials based on an iron powder background of theoretical lines giving reflectable agreement. It reveals also an unexpected small influence of the pvsw2ore size D on the stress S by the constant matrix yield stress Y. The reduction of D from 0.7 (mm) to 0.3 (mm) results in the increase of S by 20 (N mm^{-2}) only. A significant effect is noticed for small pores. Taahis phenomena is also observed in practical powder metallurgy.

5. Conclusions

1. The fatigue limit should be positively influenced by the increase in the value of the yield point of the matrix material, resulting in diminution of the range of plastic zones. The optimum solution would be to press plastic materials and only then strengthen it, e.g. by means of a thermochemical treatment.
2. Any action aimed at the reduction of the maximum pore size should be advantageous. The effect however is not linear and the reduction in size of the very big pores is less significant than that for small pores.
3. The influence of all treatments resulting in an increase of $KTHM$ i.e. increase of crack resistance of links between structure elements on the microscale level, should be positive.
4. It seems that the theory presented also can be applied to composite materials in the case when the destruction originates with an inclusion separation process.

Glossary of symbols

a crack length (mm)
A plastic zone range (mm)
B width of specimen (mm)
D pore diameter (mm)
H materials hardening coefficient
K stress intensity coefficient (N mm$^{-3/2}$)
K_{I}, K_{II} stress intensity coefficient for first and second mode of fracture
KIZ equivalent K_{I} coefficient for zigzag crack. (N mm$^{-3/2}$)
KTH threshold value of K (N mm$^{-3/2}$)
$KTHM$ KTH in microscale (N mm$^{-3/2}$)
L length of flat crack (mm)
Na real length of zigzag (mm)
N_f fatigue life in cycles
ΔP loading force variation (N)
RA reduction of area of sample
S loading stress (MPa)
W height of specimen (mm)
Y yield stress of matrix material (MPa)
α, β, γ coefficients in $A/D = f(S/Y)$ formula
αK stress concentration coefficient
δ, ρ, θ coefficients in $K = f(S, A/D)$ formula
ε_f, α_1 coefficients
$\Delta\varepsilon_p$ plastic strain components
λ, $\mathscr{P}$ parallel and perpendicular to crack front surface development correction coefficients
ψ surface development coefficient

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